Chapitre 2

Exercice 1 (1. à 7.)

Soit x une variable prenant les valeurs suivantes :

$$x_1 = 4 \qquad x_2 = -2 \qquad x_3 = 1 \qquad x_4 = 0 \qquad x_5 = -5$$

1. $\displaystyle\sum_{i=1}^{5} x_i = 4 + (-2) + 1 + 0 + (-5) = -2$

2. $\displaystyle\sum_{i=1}^{4} x_i^2 = 4^2 + (-2)^2 + 1^2 + 0^2 = 16 + 4 + 1 + 0 = 21$

3. $\displaystyle\sum_{i=1}^{3} 2x_i = 2 \cdot 4 + 2 \cdot (-2) + 2 \cdot 1 = 8 - 4 + 2 = 6$

4. $\displaystyle\sum_{i=2}^{4} (x_i - 2) = (-2 - 2) + (1 - 2) + (0 - 2) = -4 - 1 - 2 = -7$

5. $$\sum_{i=1}^{3} \frac{1}{x_i^2} = \frac{1}{4^2} + \frac{1}{(-2)^2} + \frac{1}{1^2}$$
$$= \frac{1}{16} + \frac{1}{4} + 1 = \frac{1 + 4 + 16}{16} = \frac{21}{16}$$

6. $$\sum_{i=3}^{4} \frac{1}{x_i + x_{i+1}} = \frac{1}{1 + 0} + \frac{1}{0 - 5} = 1 - \frac{1}{5} = \frac{4}{5}$$

7. $$\sum_{i=1}^{3} \frac{2x_i^2}{x_i} = \frac{2 \cdot (4)^2}{4} + \frac{2 \cdot (-2)^2}{(-2)} + \frac{2 \cdot (1)^2}{1}$$
$$= \frac{2 \cdot 16}{4} + \frac{2 \cdot 4}{(-2)} + \frac{2 \cdot 1}{1}$$
$$= \frac{32}{4} + \frac{8}{(-2)} + \frac{2}{1}$$
$$= 8 + (-4) + 2 = 6$$

Exercice 1 (8. à 10.)

Soit x une variable prenant les valeurs suivantes :

$$x_1 = 4 \qquad x_2 = -2 \qquad x_3 = 1 \qquad x_4 = 0 \qquad x_5 = -5$$

8. $$\sum_{i=2}^{3} \frac{-x_i^3}{2x_{i-1}} = \frac{-(-2)^3}{2 \cdot 4} + \frac{-(1)^3}{2 \cdot (-2)} = \frac{8}{8} + \frac{-1}{-4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$$

9. $$\begin{aligned} &\sum_i \left(x_i^2 - 3\right) \\ &\quad = (4^2 - 3) + ((-2)^2 - 3) + (1^2 - 3) + (0^2 - 3) + ((-5)^2 - 3) \\ &\quad = 13 + 1 + (-2) + (-3) + 22 = 31 \end{aligned}$$

10. $$\begin{aligned} &\sum_i \left(\sqrt{x_i^4} + x_i\right) = x_i^2 + x_i \\ &\quad = (4^2 + 4) + ((-2)^2 - 2) + (1^2 + 1) + (0^2 + 0) + ((-5)^2 - 5) \\ &\quad = 20 + 2 + 2 + 0 + 20 = 44 \end{aligned}$$

Exercice 2 (1. à 7.)

Soit x et y deux variables prenant les valeurs suivantes :

$$x_1 = 8 \quad x_2 = -2 \quad x_3 = 4 \quad x_4 = -5$$

$$y_1 = 2 \quad y_2 = -6 \quad y_3 = -1 \quad y_4 = 3$$

1. $\sum_{i=1}^{4} x_i = 8 + (-2) + 4 + (-5) = 5$

2. $\sum_{i=1}^{4} y_i = 2 + (-6) + (-1) + 3 = -2$

3. $\sum_{i=1}^{4} x_i^2 = 8^2 + (-2)^2 + 4^2 + (-5)^2 = 64 + 4 + 16 + 25 = 109$

4. $\sum_{i=1}^{4} y_i^2 = 2^2 + (-6)^2 + (-1)^2 + 3^2 = 4 + 36 + 1 + 9 = 50$

5. $\sum_{i=2}^{4} (x_i + y_i)$
$= ((-2) + (-6)) + (4 + (-1)) + ((-5) + 3)$
$= (-8) + 3 + (-2) = -7$

6. $\sum_{i} (x_i \cdot y_i)$
$= (8 \cdot 2) + ((-2) \cdot (-6)) + (4 \cdot (-1)) + ((-5) \cdot 3)$
$= 16 + 12 + (-4) + (-15) = 9$

7. $\sum_{i} \Big[(x_i + 1) \cdot (2 - y_i) \Big]$
$= [(8 + 1) \cdot (2 - 2)] + [(-2 + 1) \cdot (2 - (-6))]$
$\quad + [(4 + 1) \cdot (2 - (-1))] + [(-5 + 1) \cdot (2 - 3)]$
$= [9 \cdot 0] + [(-1) \cdot 8] + [5 \cdot 3] + [(-4) \cdot (-1)]$
$= 0 + (-8) + 15 + 4 = 11$

Exercice 2 (8. à 11.)

Soit x et y deux variables prenant les valeurs suivantes :

$$x_1 = 8 \quad x_2 = -2 \quad x_3 = 4 \quad x_4 = -5$$

$$y_1 = 2 \quad y_2 = -6 \quad y_3 = -1 \quad y_4 = 3$$

8. $$\sum_i \frac{x_i}{y_i} = \frac{8}{2} + \frac{-2}{-6} + \frac{4}{-1} + \frac{-5}{3}$$
$$= 4 + \frac{1}{3} + (-4) + \frac{-5}{3} = \frac{-4}{3}$$

9. $$\left(\sum_i x_i^2\right) \cdot \left(\sum_i y_i\right)^2 = 109 \cdot (-2)^2 = 109 \cdot 4 = 436$$

10. $$\sum_i \frac{1}{x_i \cdot y_i}$$
$$= \frac{1}{8 \cdot 2} + \frac{1}{(-2) \cdot (-6)} + \frac{1}{4 \cdot (-1)} + \frac{1}{(-5) \cdot 3}$$
$$= \frac{1}{16} + \frac{1}{12} + \frac{1}{-4} + \frac{1}{-15}$$
$$= \frac{1}{16} + \frac{1}{12} + \frac{-1}{4} + \frac{-1}{15}$$
$$= \frac{15}{240} + \frac{20}{240} + \frac{-60}{240} + \frac{-16}{240} = \frac{-41}{240}$$

11. $$\sum_i \frac{x_i}{y_i^2} = \frac{8}{(2)^2} + \frac{-2}{(-6)^2} + \frac{4}{(-1)^2} + \frac{-5}{(3)^2}$$
$$= \frac{8}{4} + \frac{-2}{36} + \frac{4}{1} + \frac{-5}{9}$$
$$= 2 + \frac{-1}{18} + 4 + \frac{-5}{9}$$
$$= 6 + \frac{-1}{18} + \frac{-10}{18}$$
$$= \frac{108 - 1 - 10}{18} = \frac{97}{18}$$

Exercice 3

Soit x et n deux variables prenant les valeurs suivantes :

$$x_1 = -3 \qquad x_2 = 9 \qquad x_3 = -1 \qquad x_4 = -6$$

$$n_1 = 2 \qquad n_2 = 5 \qquad n_3 = 0 \qquad n_4 = 8$$

1. $$\begin{aligned}\sum_i (x_i)^2 &= (-3)^2 + 9^2 + (-1)^2 + (-6)^2 \\ &= 9 + 81 + 1 + 36 \\ &= 127\end{aligned}$$

2. $$\begin{aligned}\left(\sum_i x_i\right)^2 &= (-3 + 9 - 1 - 6)^2 \\ &= (-1)^2 \\ &= 1\end{aligned}$$

3. $$\begin{aligned}\sum_i (x_i) \cdot \sum_i (n_i) &= (-3 + 9 - 1 - 6) \cdot (2 + 5 + 0 + 8) \\ &= (-1) \cdot 15 \\ &= -15\end{aligned}$$

4. $$\begin{aligned}\sum_i x_i n_i &= (-3) \cdot 2 + 9 \cdot 5 + (-1) \cdot 0 + (-6) \cdot 8 \\ &= -6 + 45 + 0 - 48 \\ &= -9\end{aligned}$$

Exercice 4

Exprime les sommes suivantes avec la notation de sommation :

1. $x_1 + x_2 + x_3 + x_4 + \cdots + x_{18} = \sum_{i=1}^{18} x_i$

2. $\frac{x_1}{y_1} + \frac{x_2}{y_2} + \frac{x_3}{y_3} + \frac{x_4}{y_4} + \cdots + \frac{x_{100}}{y_{100}} = \sum_{i=1}^{100} \frac{x_i}{y_i}$

3. $a_1^2 b_1 + a_2^2 b_2 + a_3^2 b_3 + a_4^2 b_4 + \cdots + a_9^2 b_9 = \sum_{i=1}^{9} a_i^2 b_i$

4. $a_1^2 b_1 + a_2^2 b_1 + a_3^2 b_1 + a_4^2 b_1 + \cdots + a_9^2 b_1 = \sum_{i=1}^{9} a_i^2 b_1$

5. $\frac{x_1}{y_1^2} + \frac{x_2}{y_2^2} + \frac{x_3}{y_3^2} + \frac{x_4}{y_4^2} + \cdots + \frac{x_{12}}{y_{12}^2} = \sum_{i=1}^{12} \frac{x_i}{y_i^2}$